Uitwerking tentamen Basiswiskunde januari 2007

1 $$\frac{1}{(x-4)(x+4)}-\frac{2}{3(x-4)}=\frac{3-2(x+4)}{3(x-4)(x+4)}=\frac{-2x-5}{3(x-4)(x+4)}$$

2 $$(x^4-1)(x^2-x-6)=(x^2+1)(x+1)(x-1)(x+2)(x-3)$$

3 $$4x-1=0\vee 4x-1=1\vee 4x-1=-1 \text{ dus } x=\tfrac{1}{4}\vee x=\tfrac{1}{2}\vee x=0$$

4 $$(2x)^3+3\cdot(2x)^2\cdot -3+3\cdot 2x\cdot(-3)^2+(-3)^3=8x^3-36x^2+54x-27$$

5 $$\tfrac{1}{2}\cdot 16\cdot(5\tfrac{1}{2}+13)=148$$

6 $$D_{\arctan x}=\mathbb{R} \Rightarrow \sqrt{x}\in\mathbb{R} \Rightarrow x\geq 0$$

$$y'=\frac{1}{1+(\sqrt{x})^2}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2(x+1)\sqrt{x}}$$

7 $$y'=\frac{3x^2}{x^3+2} \qquad y''=\frac{(x^3+2)\cdot 6x-3x^2\cdot 3x^2}{(x^3+2)^2}=\frac{-3x^4+12x}{(x^3+2)^2}$$

$$y''=0\Rightarrow x^4-4x=0\Rightarrow x(x^3-4)=0\Rightarrow x=0\vee x=\sqrt[3]{4}$$

8 $$F(x)=\frac{1}{\ln 2}2^x-\tfrac{1}{5}\cos 5x+C$$

9 $$f(x)=3(5x+2)^{-\frac{1}{2}}+\frac{1}{3x+10} \text{ dus } F(x)=\tfrac{6}{5}\sqrt{(5x+2)}+\tfrac{1}{3}\ln|3x+10|+C$$

10 $$2\cos^2 x-1=\cos^2 x\Rightarrow\cos^2 x=1\Rightarrow\cos x=\pm 1 \text{ dus } x=0\vee x=\pi\vee x=2\pi$$

11 $$\cos 2x=2\cos^2 x-1\Rightarrow\tfrac{1}{2}\sqrt{3}=2\cos^2\tfrac{1}{12}\pi-1\Rightarrow\cos^2\tfrac{1}{12}\pi=\tfrac{1}{2}+\tfrac{1}{4}\sqrt{3}$$

$$\cos\tfrac{1}{12}\pi=\sqrt{\tfrac{1}{2}+\tfrac{1}{4}\sqrt{3}}\Rightarrow\cos(-\tfrac{1}{12}\pi)=\sqrt{\tfrac{1}{2}+\tfrac{1}{4}\sqrt{3}}$$

$$\cos 2x=1-2\sin^2 x\Rightarrow\tfrac{1}{2}\sqrt{3}=1-2\sin^2\tfrac{1}{12}\pi\Rightarrow\sin^2 x=\tfrac{1}{2}-\tfrac{1}{4}\sqrt{3}$$

$$\sin\tfrac{1}{12}\pi=\sqrt{\tfrac{1}{2}-\tfrac{1}{4}\sqrt{3}}\Rightarrow\sin(-\tfrac{1}{12}\pi)=-\sqrt{\tfrac{1}{2}-\tfrac{1}{4}\sqrt{3}}$$

12 $(z-5)^2 - 25 + 32 = 0 \Rightarrow (z-5)^2 = -7$

$z - 5 = \pm i\sqrt{7} \Rightarrow z = 5 \pm i\sqrt{7}$

13 $|z| = 2\sqrt{2}$ en $\arg z = \frac{3}{4}\pi$

$(2\sqrt{2}e^{\frac{3}{4}\pi i})^5 = 128\sqrt{2}e^{\frac{15}{4}i} = 128\sqrt{2}e^{-\frac{1}{4}\pi i}$

$= 128\sqrt{2}(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = 128 - 128i$

14 De lesstof was toen anders. Deze opgave is niet van toepassing voor de cursus 2008-2009

Stel $z = x + iy$ dan $\sqrt{(x^2 + (y-3)^2} = \sqrt{(4-x)^2 + (1-y)^2}$ dus

$x^2 + y^2 + 6y + 9 = 16 - 8x + x^2 + 1 - 2y + y^2$ is equivalent met

$y = 2x - 2$ in $\mathbb{R}^2$

15 $n = 2 \Rightarrow 2^2 = \frac{1}{6} * 2 * 3 * 5 - 1 = 4$ klopt

$\sum_{k=2}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2 = \frac{1}{6}n(n+1)(2n+1) - 1 + (n+1)^2 = (n+1)(\frac{1}{6}(2n+1) + n + 1) - 1 =$

$(n+1)(\frac{1}{3}n^2 + \frac{7}{6}n + 1) - 1 = (n+1) * \frac{1}{6}(2n^2 + 7n + 6) - 1 = \frac{1}{6}(n+1)(n+2)(2n+3) - 1$

Klopt!