Beknopte uitwerking 31-3-2005 Lin. Alg. S/F/M

1. a. Bijvoorbeeld $n = \begin{pmatrix}1\\0\\1\end{pmatrix} \times \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$

b. Los t, s, r op uit

$$\begin{pmatrix}2\\0\\3+t\end{pmatrix} = \begin{pmatrix}1+r\\3+s\\r\end{pmatrix}$$ dus: $r=1$, $s=-3$. Snijpunt $\begin{pmatrix}2\\0\\1\end{pmatrix}$

c. $\cos\alpha = \left| \dfrac{n \cdot \begin{pmatrix}0\\0\\1\end{pmatrix}}{\|n\| \cdot \left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|} \right| = \dfrac{1}{\sqrt{2}}$. Dus $\alpha = \dfrac{\pi}{4}$.

De hoek tussen V en l is dan ook $\frac{\pi}{4}$.

2.a $$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 7 & 3 & 0 & 1 & 0 \\ 2 & 4 & 4 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 2 & -2 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 2 & 0 & -1/2 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1/2 \end{array}\right) \to$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -2 & -1/2 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1/2 \end{array}\right)$$. Dus de inverse is $\begin{pmatrix} 8 & -2 & -1/2 \\ -3 & 1 & 0 \\ -1 & 0 & 1/2 \end{pmatrix}$

b. De oplossing is $\begin{pmatrix} 8 & -2 & -1/2 \\ -3 & 1 & 0 \\ -1 & 0 & 1/2 \end{pmatrix}\begin{pmatrix}2\\6\\2\end{pmatrix} = \begin{pmatrix}3\\0\\-1\end{pmatrix}$.

3a. $1-y$, $(1-y)^2$, $(1-y)^3$ b. $(1-y)^{n-1}$

4a. $\lambda^2 + 6\lambda + 25 = (\lambda+3)^2 + 16$. $\lambda = -3 \pm 4i$. Dus

$y(x) = c_1 e^{-3x}\cos 4x + c_2 e^{-3x}\sin 4x \qquad c_1, c_2 \in \mathbb{R}$

b. Probeer $y = a\cos x + b\sin x$. Je vindt:

$(-6a + 24b)\sin x + (6b + 24a)\cos x = 102\cos x$ dus

$-6a + 24b = 0$, $24a + 6b = 102 \qquad a = 4$, $b = 1$

Dus $y = 4\cos x + \sin x + c_1 e^{-3x}\cos 4x + c_2 e^{-3x}\sin 4x$.

c. $y(0) = 0$ geeft ~~$c_1 + c_2 \ldots$~~ $c_1 = 0$

$y'(x)$ wordt dan $-3c_2 e^{-3x}\sin 4x + 4c_2 e^{-3x}\cos 4x$.

Dus $y'(0) = 4c_2$ en $c_2 = \frac{7}{4}$. Dus $y(x) = \frac{7}{4} e^{-3x}\sin 4x$

5a. $A = \begin{pmatrix} -3 & -2 & 0 \\ -3 & -3 & -3 \\ 0 & -1 & -3 \end{pmatrix}$. Eigenwaarden: $-3, -6, 0$

Eigenvectoren, resp. $\begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\3\\1\end{pmatrix}, \begin{pmatrix}2\\-3\\1\end{pmatrix}$. Dus:

$$\begin{pmatrix}x(t)\\y(t)\\z(t)\end{pmatrix} = c_1 e^{-3t}\begin{pmatrix}1\\0\\-1\end{pmatrix} + c_2 e^{-6t}\begin{pmatrix}2\\3\\1\end{pmatrix} + c_3\begin{pmatrix}2\\-3\\1\end{pmatrix}$$

b. $c_1 = 1/3$, $c_2 = 1/2$ $c_3 = -1/6$.